



Making Room For More

In mathematics there is a paradox known as the Banach-Tarski paradox, it proposes that if you take an object and split it into five pieces, by simply rearranging these five pieces you can turn that one object into two exact copies of the original object. Mathematically this can be proven, but in the real world you cannot create something from nothing. However, there is a cool demonstration that you can do at home to see this principal in action! One thing to keep in mind is that you're not actually creating something from nothing, the pieces of that something are in reality coming from the original object, but as it's spread over a larger area it becomes almost unnoticeable. This creates the illusion of our paradox!

Materials

- Scissors
- Pen or Pencil
- Ruler
- Paper
- Printer (optional)

1 The first step is to use your paper to draw your "chocolate bar", if you have a printer you can just print out any 5x5 chocolate bar picture and use that as the base, this process should work regardless of whether you use a square or rectangular bar shape.

2 The second step is to divide your bar into five pieces, to make this easier to repeat, we'll divide our chocolate bar into a grid. Each row we'll represent with a letter, from A-E, and each column will be numbered, 1-5.

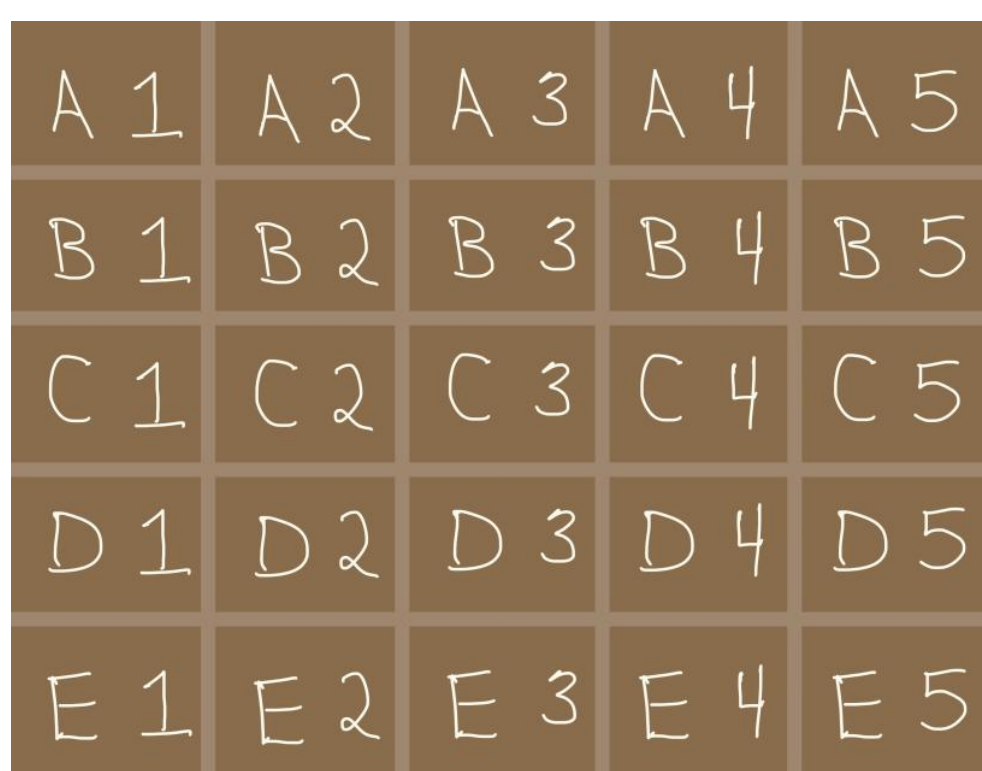
3 Cutting diagonally from about $\frac{2}{3}$ - $\frac{3}{4}$ of the way along bar A2 across to about $\frac{1}{4}$ - $\frac{1}{3}$ of the way along bar E4. From there separate the two halves, putting the left half aside, that section is complete, we'll call that section 1. Take the right half of your bar, and cut along the top of row D. Separate out the bottom section you just cut off, and put it aside, we'll refer to this section as section 3. Next take the remaining top section, and cut along the border between column 4 and 5, take the left side of that and put that aside, we will call that section 2. Last, take the three blocks you have remaining, cut A5 off of top of the remaining section. We will call the individual block A5 section 5, and the other part section 4.

MOST*

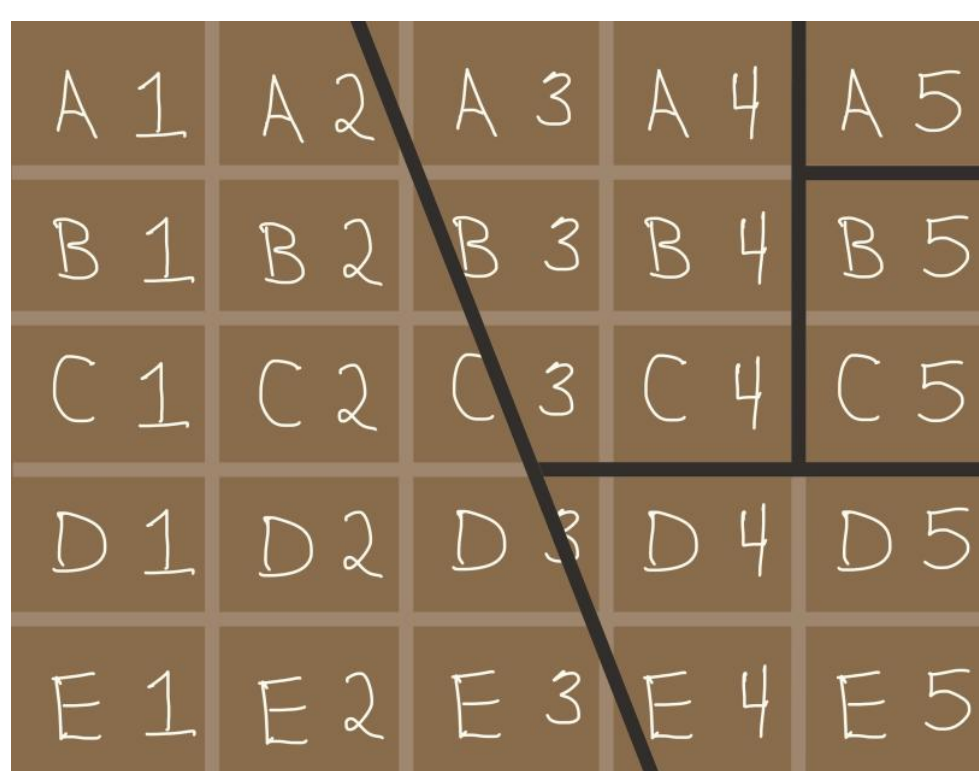
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4 Next, take section 5 and pull it aside. It's time to start reassembling our bar! First place section 1 down, to the right of it, aligned with the bottom of the bar reinsert section 2. Above section 2 and along the right edge of section 1 insert section 3. Finally, insert section 4 into the remaining space to the right of section 3 and above section 2.

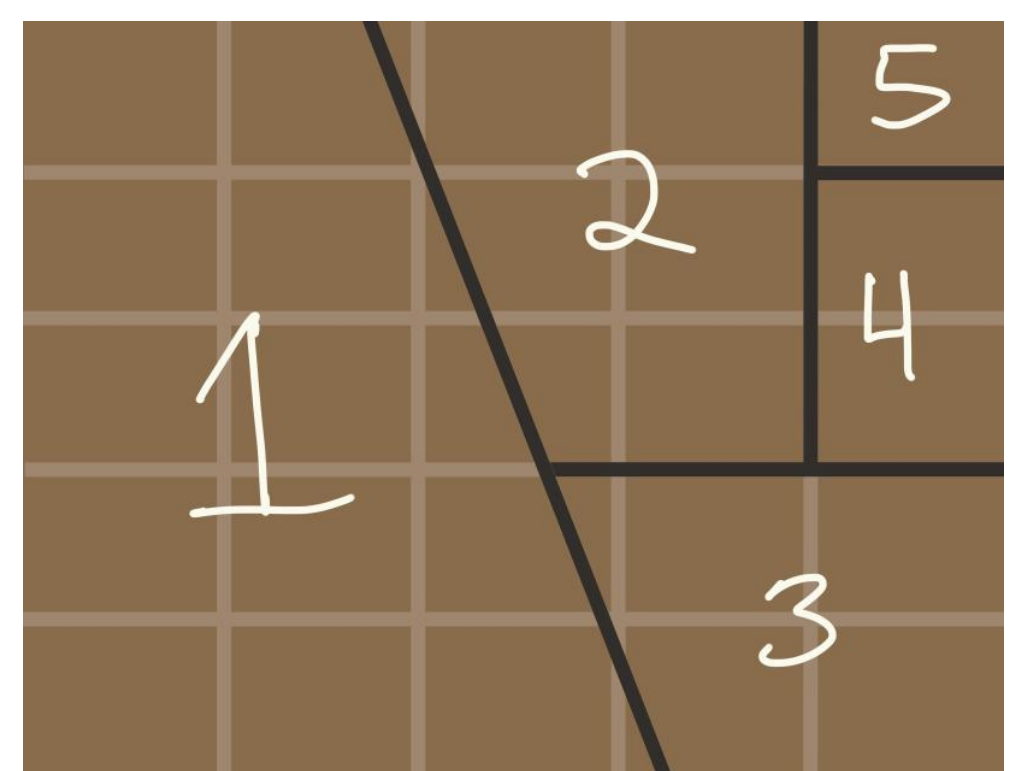
5 Now you should have a complete 5x5 chocolate bar again, but notice you now have an extra piece! As stated earlier, you're not really creating something for nothing, but rather you've just made the illusion of creating something from nothing! For a little more math fun, try measuring the length and height of the chocolate bar before and afterwards, see if the change in area from before to afterwards matches the area of your extra piece of chocolate!



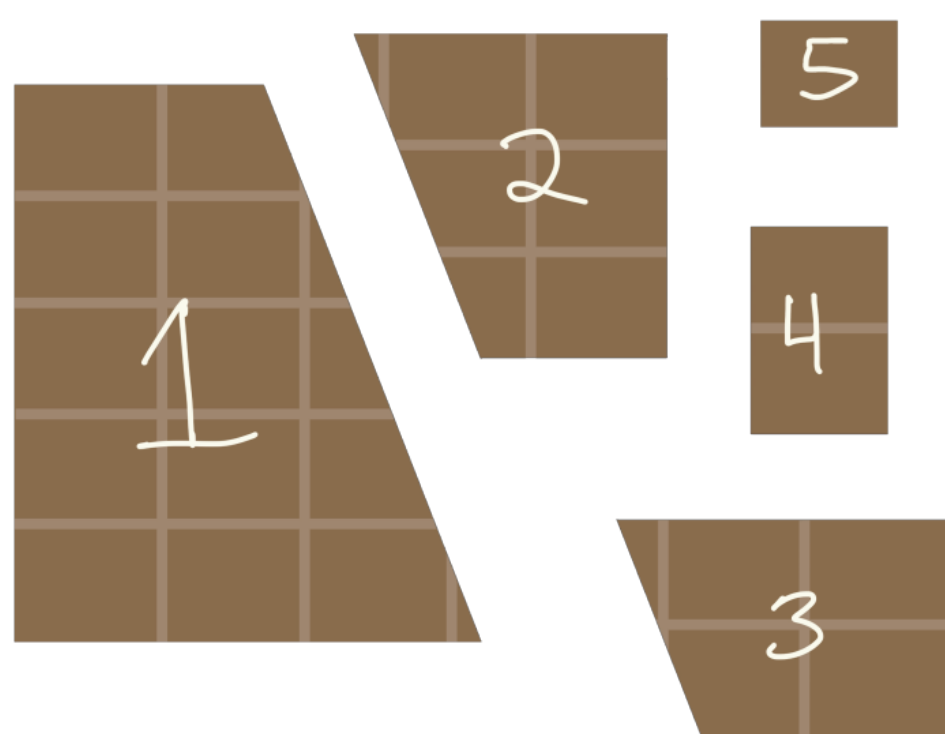
Step 2



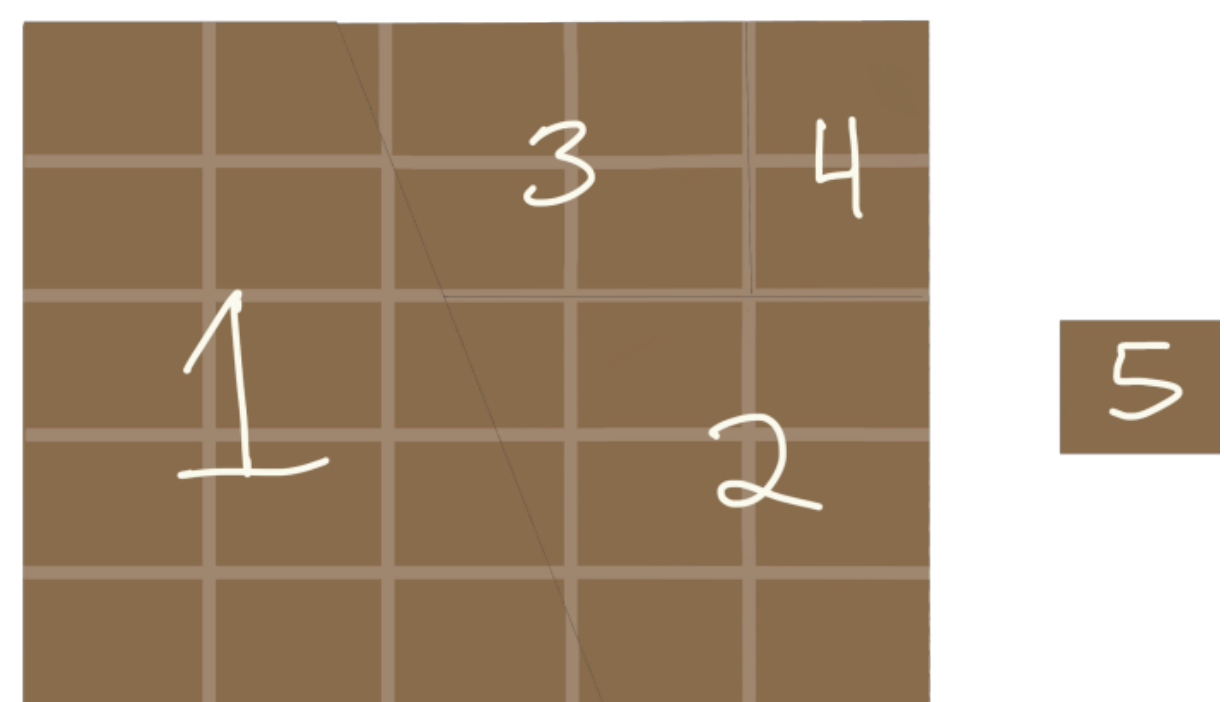
Step 3



Step 3



Step 4



Step 5